Minimum Drag Bodies of Given Length and Base Using Newtonian Theory

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Theme

THE forebody of a high-speed aircraft or missile generates a substantial proportion of the overall drag, such that even a modest reduction in the forebody drag can produce a significant improvement in the performance. The requirement that the forebody blend into a fuselage which is nearly axisymmetric has resulted in an intuitive assumption that any low-drag forebody will also be nearly axisymmetric. Indeed, this assumption is built into most optimization studies[1] by geometric constraints on the forebody. Even when the forebody cross-sections are left unspecified, it is usually assumed that these sections and those of the afterbody are similar, and this approach has led only to unrealistic star cross-section bodies which are discussed in Ref. 1. In this paper, although the base contour is fixed, the cross-sections upstream of the base are merely required to be convex. The minimum drag shapes which emerge using these constraints are termed "spatular" shapes, because of their flat region near the nose and near circular cross-section downstream. The shapes combine the prospects of significantly lower drag (10-15% lower) and improved pilot visibility.

Contents

Minimum drag bodies are found within the following constraints: 1) the surface pressure is given by Newtonian theory (i.e. $C_p \propto \cos^2\delta$ where δ is the angle of inclination of the surface normal to the freestream); 2) the base shape is given; 3) cross-sections through the body normal to the stream have convex contours; and 4) body length or body length and surface area are given.

The minimum drag body with surface area given and length free has previously been shown[1] to have δ constant over the surface, such that for a circular base the minimum drag body is a cone. With length restricted however, the minimum drag axisymmetric body derived by Newton[2] has less drag than the cone, its drag being reproduced here as a fraction of that of the cone with the same thickness parameter (τ) by curve (1) of Fig. 1.

Newton's axisymmetric constraint is more restrictive than constraint 3, which permits the investigation of shapes such as the spatular shape of Fig. 2. Unfortunately the drag of these shapes generally has to be found by numerical integration of the pressure over the surface. However a simple spatular shape which can be investigated analytically is the body formed by drawing tangents to a cone from two streamlines passing through opposite ends of a base diameter. The equation of the body surface is then

$$r = x\tau/2 \text{ for } x \leq |\sin\theta|$$

$$r = \tau/(2|\cos\theta|\,(1/x^2 - 1)^{1/2} + 2|\sin\theta|) \text{ for } x \geq |\sin\theta|$$

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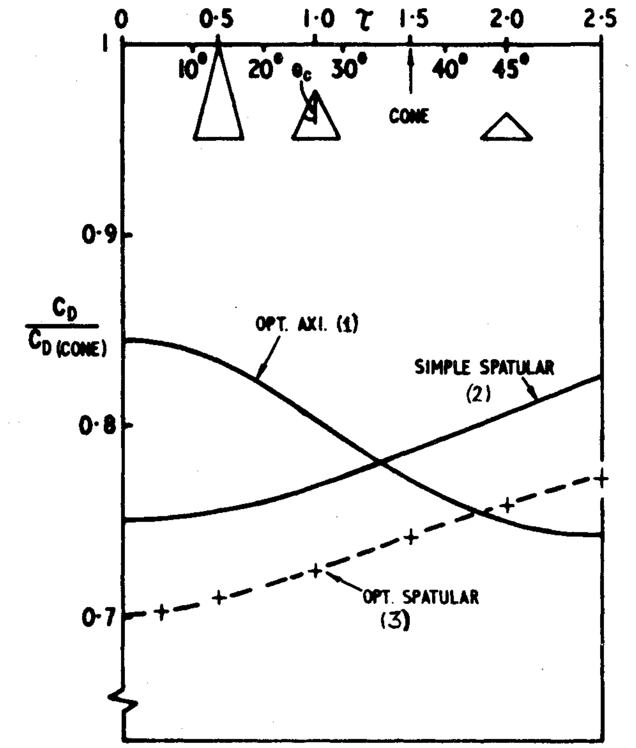

Fig. 1 Drag of bodies of given circular base diameter-to-length ratio (τ).

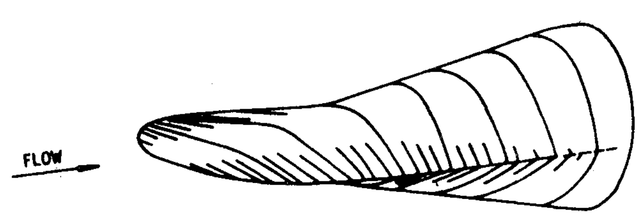

Fig. 2 Spatular body.

where x, r, and θ are cylindrical polar coordinates with x in the freestream direction, and its drag coefficient is given by

$$C_D = \frac{\tau^2}{4+\tau^2} + 1 - \frac{4}{\tau^2}\log\left(1 + \frac{\tau^2}{4}\right) \tag{1}$$

Equation (1) is plotted as curve 2 in Fig. 1, and it can be seen that this simple spatular body has less drag than Newton's body for $\tau = 1.33$.

To reduce the drag of spatular bodies further, a numerical optimization process is used in which local modifications to the shape are progressively evaluated and retained if they reduce the drag. Sections through a typical optimum shape which has $\tau = 0.5$ are shown in Fig. 3, and the drag coefficient of the optimized bodies for various values of τ are indicated by the crosses on curve 3 of Fig. 1. For values of τ of most

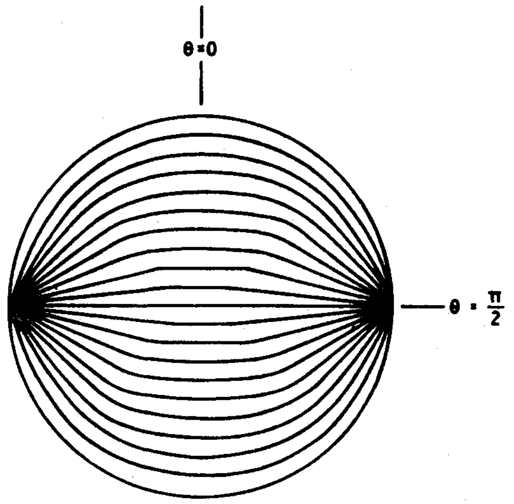

Fig. 3 Sections through the minimum drag spatular body with $\tau = 0.5$. $C_D = 0.0836$. $S/B = 5.91$.

interest (i.e. $\tau \simeq 0.5$), the drag of the optimum spatular body is 15% less than the drag of the optimum axisymmetric body. Similar improvement in the drag is found for bodies with noncircular bases, and, in particular for elliptic bases, the drag is found to vary approximately in proportion to the ratio of the minor and major axes.

A disadvantage of spatular bodies is their large wetted area compared with axisymmetric bodies. For example, for the spatular body shown in Fig. 3, the wetted-area to base-area ratio (S/B) is 5.91. This can be compared with 4.8 for the optimum axisymmetric body and 4.12 for the cone with the same value of τ. The addition of a surface area constraint thus favors axisymmetric bodies. In Fig. 4 optimum bodies are considered with both a length ($\tau = 0.5$) and a surface area constraint (S/B given). The most important feature of Fig. 4 is the small increase in the drag of spatular bodies for spatular bodies for values of S/B down to about 5. This wetted area decrease is achieved by the flattened region near the nose becoming slightly smaller and assuming a more rounded profile as illustrated in Fig. 2 and indicated in Fig. 4. For $S/B < 5$ the drag is significantly above the free surface area minimum, and when S/B is 4.8 the minimum drag body can be either of two very different shapes, one axisymmetric and the other spatular. For $S/B < 4.8$ the optimum bodies are axisymmetric.

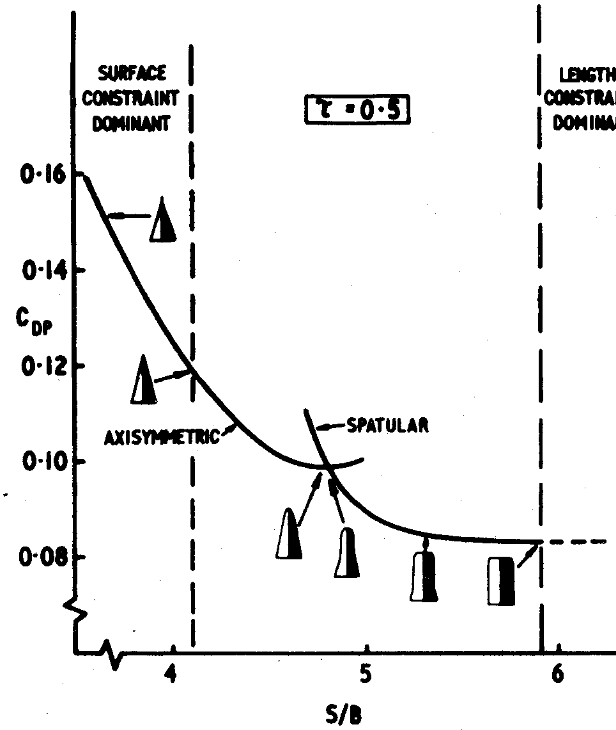

Fig. 4 Drag coefficients of optimum noses with $\tau = 0.5$ and both length and surface area constraints.

To consider the effects of friction drag, it is taken as a first approximation to be proportional to the wetted area [i.e. $C_{DF} = C_F \times (S/B)$]. The optimum body of minimum total drag occurs at the point on the curve which has a gradient of $-C_F$. For practical values of skin friction spatular bodies are found to be optimum; for slender bodies ($\tau \leq 0.2$) or large skin friction the optimum bodies are axisymmetric.

References

[1] Miele, A., "Theory of Optimum Aerodynamics Shapes," *Applied Mathematics and Mechanics,* Vol. 9, Academic Press, New York and London, 1965.

[2] Newton, I., "Mathematical Principles of Natural Philosophy," A. Motte's translation revised by F. Cajori, University of California Press, Berkeley, Calif., 1934.

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